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1 Lattice Yang-Mills

1. Read about the random current representation for the Ising model (which involves spins of ± 1 on vertices — you can think of these as 1×1 orthogonal matrices if you like) and see whether you can describe something similar for higher dimensional matrix gauge theories.

2. Derive something about large N asymptotics of the number of pairs (M, f) where N is a planar map and f is a graph homomorphism from M into some fixed graph (maybe a lattice, maybe some smaller graph). Start by reading some of the overview works of Eynard about multi-matrix models and see if you find something you can generalize or extend to a different setting.

3. Read Chatterjee's $SO(N)$ paper and understand and explain the hard part of the calculation in a short survey article.

4. Extend the results of Chatterjee and Jafarov (for $SO(N)$) to another matrix group, or to another lattice or graph. Any extension would be interesting.

5. Derive Chatterjee's result a different way: with Wick's theorem and approximation limits.

6. Categorize functions F that satisfy basic consistency.

7. Produce continuum version in Gaussian case. Start with Gaussian model on space of connections. Then consider a continuum of "plaquettes."

8. Understand surface stories for Yang-Mills in 0, 1, and 2 dimensionals.

9. Try to read "Gaussian asymptotics of discrete β-ensembles" or another work treating point process point of view for random eigenvalues or non-intersecting paths.

10. Read about Lindström-Gessel-Viennot a.k.a. Karlin-McGregor (the determinantal formula for eigenvalue distribution as derived from non-intersecting paths, Dyson's Brownian motion, Vandermone determi-

nants, etc.) which gives one intuitive way to understand why determinantal point process appear in many settings involving non-intersecting paths or matrix eigenvalues. See if you can find an interesting generalization of these stories. Maybe you can investigate a multi-matrix setting or a two dimensional particle system (a matrix with complex values) in some way.

11. Explain surface boundary conditions and corresponding gauge stories.

12. Liouville quantum gravity: better understand what is meant by the $c > 1$ versions of the theory. Prove that surfaces sampled from this theory, conditioned to be finite, converge to continuum random trees in an appropriate scaling limit.

13. Build continuum LQG based gauge theory.

2 Mismatch

1. Describe the left and right boundary length processes corresponding to space-filling SLE'_κ drawn on top of γ-LQG when $\gamma \neq \kappa^2$.

2. What about non-space-filling $\mathrm{SLE}_{\kappa'}$?

3. Consider a $\sqrt{8/3}$-LQG decorated by 2 instances of the GFF and 1 spanning tree and try to show that when you follow one spanning tree branch, conditional law of two sides *given* the field/ values on branch itself are independent. There are many variants of this question (using different kinds of decorations, maybe Brownian loop soups in place of GFF instances, etc.) that boil down to asking for some conditional independence of two sides given something on the boundary itself.

4. Try to understand Markov property for evolving strings in $c > 1$ LQG. Describe the time reversal of the evolving string law and give an analog of a Wilson loop expression for sufficiently regular strings.

5. Consider the object you get by gluing together a disk and a CRT (see Lin and Rohde). Can this be related to a QLE-type growth model or something similar? What about an infinite version?

3 Mating of trees and space-filling SLE

1. Consider a stable Lévy process L_t (indexed by $\mathbb{R}$, defined modulo additive constant) with only positive jumps and with parameter $\alpha \in (0,1)$. Let $J_1, J_2, \ldots$ be an enumeration of the jumps. For each vertical jump line segment J_i, which occurs at a time t_i, let T_i denote the next subsequent time after t_i at which the process $L.$ reaches the value of L before the jump (i.e., the left limit of $L.$ at t_i). Then denote by S_i the set of times at which L (restricted to $[t_i, T_i]$) achieves a minimum value. That is,
$$S_i = \{t \in [t_i, T_i] : s \in [t_i, t] \text{ implies } L_s \leq S_t\}.$$
The collection $\{S_i\}$ is a random countable collection of pairwise disjoint closed subsets of $\mathbb{R}$ that are "nested" in a certain sense. Now suppose that $\{\tilde{S}_i\}$ is an independent instance of this random collection. Declare an element of $\{S_i\}$ and an element of $\{\tilde{S}_i\}$ to be adjacent if they intersect. Now we have a random bipartite graph. Is this bipartite graph almost surely connected? This problem is equivalent to a problem about the connectedness of graph of complementary components of a self-intersecting but not space-filling SLE curve (two components connected if boundaries intersect).

2. Could we at least prove connectedness if we had k independent instances of $\{S_i\}$ (for some sufficiently large finite value k) instead of 2?

3. Mullin's bijection implies that uniform-spanning-tree-decorated random planar maps scale have a certain scaling limit in the peanosphere topology. Can this be generalized to UST-decorated quadrangulations, UST-decorated triangulations, or other types of UST-decorated discrete graphs?

4. Can one further generalize the peanosphere convergence for FK models (as derived via the hamburger-cheeseburger bijection) to triangulations, quadrangulations, etc.?

5. In hamburger cheeseburger setting, suppose we choose independent instances of FK given planar map. Can one show that in limit of the coupling the surface is the same with curves different.

6. Explain Brownian motion results one obtains as consequence of mating of trees (alternating directions, getting coupling of flow line and field, what path determined by field means on Brownian motion side).

7. Can one put metric on peanosphere when $\gamma^2 \neq 8/3$? Read work by Gwynne, Holden, Sun about exponent existence.

4 Liouville Quantum Gravity Properties

1. Understand the torus (see works by Rhodes, Vargas and others on LQG side). The LQG torus defined via resampling. The LQG torus defined in terms of some Laplacian determinant. The Brownian map torus. The peanosphere torus. How are they all defined and how are they all related? Can something be said about the relationship at least in the $\gamma^2 = \kappa = 2$ UST case? Or the $\gamma^2 = \kappa = 8/3$ case?

5 KPZ growth

1. Discrete KPZ: another interpolation between (-1)-DBM and 0-DBM

2. KPZ holds for KPZ: that is, one can consider the SRW measure on paths, then weight randomly by white noise to get a new measure, and the new measure satisfies a KPZ. Take distance between two walks to be 2^{-k} where k is last point at which they agree. You can think about covering your random subset of the set of all paths — either fixed radius in this standard metric, or of fixed volume in the random measure.

3. Can one characterize KPZ growth by scale invariance and local independence?

6 Other SLE stories

1. Consider SLE_6 on universal cover of complement of grid of points. This process has a boundary that looks like SLE_6 boundary, but in another sense it approximates Brownian motion. What good are these sorts of "in between Brownian motion and SLE_6" objects?

7 Other interesting models

1. Scaling limits for tricolor percolation

2. Dynamical triangulations and causal quantum gravity.

3. Understand the Gaussian field arising from commutative Yang Mills theory (electromagnetism). (Talk to Pfeffer et al about what is known thus far.)